

問題

(1)

1	(1)	$\exists x(A(x) \vee B(x))$	H.
2	(2)	$A(x) \vee B(x)$	H.
3	(3)	$A(x)$	H.
3	(4)	$\exists xA(x)$	3, \exists -I
3	(5)	$\exists xA(x) \vee \exists xB(x)$	4, \vee -I
6	(6)	$B(x)$	H.
6	(7)	$\exists xB(x)$	6, \exists -I
6	(8)	$\exists xA(x) \vee \exists xB(x)$	7, \vee -I
2	(9)	$\exists xA(x) \vee \exists xB(x)$	2,3-5,6-8, \vee -E
1	(10)	$\exists xA(x) \vee \exists xB(x)$	1,2-9, \exists -E
	(11)	$\exists x(A(x) \vee B(x)) \rightarrow (\exists xA(x) \vee \exists xB(x))$	1-10, \rightarrow -I

(2)

1	(1)	$\forall x(A(x) \rightarrow B(x))$	H.
2	(2)	$\forall xA(x)$	H.
1	(3)	$A(x) \rightarrow B(x)$	1, \forall -E
2	(4)	$A(x)$	2, \forall -E
1,2	(5)	$B(x)$	3,4, \rightarrow -E
1,2	(6)	$\forall xB(x)$	5, \forall -I
1	(7)	$\forall xA(x) \rightarrow \forall xB(x)$	2-6, \rightarrow -I
	(8)	$\forall x(A(x) \rightarrow B(x)) \rightarrow (\forall xA(x) \rightarrow \forall xB(x))$	1-7, \rightarrow -I

(3)

1	(1)	$\forall xA(x)$	H.
2	(2)	$\exists x\neg A(x)$	H.
3	(3)	$\neg A(x)$	H.
1	(4)	$A(x)$	1, \forall -E
1,3	(5)	\perp	3,4, \neg -E
1,2	(6)	\perp	2,3-5, \exists -E
1	(7)	$\neg\exists\neg A(x)$	2-6, \neg -I
	(8)	$\forall xA(x) \rightarrow \neg\exists\neg A(x)$	1-7, \rightarrow -I

(4)

1	(1)	$\exists x(A(x) \wedge B)$	H.
2	(2)	$A(x) \wedge B$	H.
2	(3)	$A(x)$	2, \wedge -E
2	(4)	$\exists xA(x)$	3, \exists -I
2	(5)	B	2, \wedge -E
2	(6)	$\exists xA(x) \wedge B$	4,5, \wedge -I
1	(7)	$\exists xA(x) \wedge B$	1,2-6, \exists -E
	(8)	$\exists x(A(x) \wedge B) \rightarrow (\exists xA(x) \wedge B)$	1-7, \rightarrow -I
9	(9)	$\exists xA(x) \wedge B$	H.
9	(10)	$\exists xA(x)$	9, \wedge -E
11	(11)	$A(x)$	H.
9	(12)	B	9, \wedge -E
9,11	(13)	$A(x) \wedge B$	11,12, \wedge -I
9,11	(14)	$\exists x(A(x) \wedge B)$	13, \exists -I
9	(15)	$\exists x(A(x) \wedge B)$	11, 12-14, \exists -E
	(16)	$(\exists xA(x) \wedge B) \rightarrow \exists x(A(x) \wedge B)$	9-15, \rightarrow -I
	(17)	$\exists x(A(x) \wedge B) \leftrightarrow (\exists xA(x) \wedge B)$	8,16, \leftrightarrow -定義

(5)

1	(1)	$\exists xA(x)$	H.
2	(2)	$A(x)$	H.
3	(3)	$\forall x\neg A(x)$	H.
3	(4)	$\neg A(x)$	3, \forall -E
2,3	(5)	\perp	2,4, \neg -E
2	(6)	$\neg\forall\neg A(x)$	3-5, \neg -I
1	(7)	$\neg\forall\neg A(x)$	1,2-6, \exists -E
	(8)	$\exists xA(x) \rightarrow \neg\forall\neg A(x)$	1-7, \rightarrow -I

(6) 省略

(7)

1	(1)	$\neg\forall yF(y)$	H.
2	(2)	$\neg\exists x\neg F(x)$	H.
3	(3)	$\neg F(x)$	H.
3	(4)	$\exists x\neg F(x)$	3, \exists -I
2,3	(5)	\perp	2,4, \neg -E
2	(6)	$\neg\neg F(x)$	3-5, \neg -I
2	(7)	$F(x)$	6, $\neg\neg$ -E
2	(8)	$\forall yF(y)$	7, \forall -I
1,2	(9)	\perp	1,8, \neg -E
1	(10)	$\neg\neg\exists x\neg F(x)$	2-9, \neg -I
1	(11)	$\exists x\neg F(x)$	10, $\neg\neg$ -E
	(12)	$\neg\forall yF(y) \rightarrow \exists x\neg F(x)$	1-11, \rightarrow -I

(8)

1	(1)	$\exists x(A \rightarrow B(x))$	H.
2	(2)	$A \rightarrow B(x)$	H.
3	(3)	A	H.
2,3	(4)	$B(x)$	2,3, \rightarrow -E
2,3	(5)	$\exists xB(x)$	4, \exists -I
2	(6)	$A \rightarrow \exists xB(x)$	3-5, \rightarrow -I
1	(7)	$A \rightarrow \exists xB(x)$	1,2-6, \exists -E
	(8)	$\exists x(A \rightarrow B(x)) \rightarrow (A \rightarrow \exists xB(x))$	1-7, \rightarrow -I

(9)

1	(1)	$A \rightarrow \exists xB(x)$	H.
2	(2)	A	H.
1,2	(3)	$\exists xB(x)$	1,2, \rightarrow -E
4	(4)	$B(x)$	H.
1,4	(5)	$A \rightarrow B(x)$	2-4, \rightarrow -I
1,4	(6)	$\exists x(A \rightarrow B(x))$	5, \exists -I
1	(7)	$\exists x(A \rightarrow B(x))$	3, 4-6, \exists -E
	(8)	$(A \rightarrow \exists xB(x)) \rightarrow \exists x(A \rightarrow B(x))$	1-7, \rightarrow -I

(10)

1	(1)	$\exists x(A(x) \rightarrow B(x))$	H.
2	(2)	$A(x) \rightarrow B(x)$	H.
3	(3)	$\forall xA(x)$	H.
3	(4)	$A(x)$	3, \forall -E
2,3	(5)	$B(x)$	2,4, \rightarrow -E
2,3	(6)	$\exists xB(x)$	5, \exists -I
2	(7)	$\forall xA(x) \rightarrow \exists xB(x)$	3-6, \rightarrow -I
1	(8)	$\forall xA(x) \rightarrow \exists xB(x)$	1,2-7, \exists -E
	(9)	$\exists x(A(x) \rightarrow B(x)) \rightarrow (\forall xA(x) \rightarrow \exists xB(x))$	1-8, \rightarrow -I

(11) 省略

(12)

1	(1)	$\exists x\neg F(x)$	H.
2	(2)	$\neg F(x)$	H.
3	(3)	$\forall xF(x)$	H.
3	(4)	$F(x)$	3, \forall -E
2,3	(5)	\perp	2,4, \neg -E
2	(6)	$\neg\forall xF(x)$	3-5, \neg -I
1	(7)	$\neg\forall xF(x)$	1,2-6, \exists -E
	(8)	$\exists x\neg F(x) \rightarrow \neg\forall xF(x)$	1-7, \rightarrow -I

(13)

1	(1)	$\forall x(F(x) \wedge G(x))$	H.
1	(2)	$F(x) \wedge G(x)$	1, \forall -E
1	(3)	$F(x)$	2, \wedge -E
1	(4)	$\forall xF(x)$	3, \forall -I
1	(5)	$G(x)$	2, \wedge -E
1	(6)	$\forall xG(x)$	5, \forall -I
1	(7)	$\forall xF(x) \wedge \forall xG(x)$	4,6, \wedge -I
	(8)	$\forall x(F(x) \wedge G(x)) \rightarrow (\forall xF(x) \wedge \forall xG(x))$	1-7, \rightarrow -I