λ Fuzzy Measure Identification Methods using λ and Weights

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1 Introduction

This document is the English summary of the paper[1] to shows the fuzzy measure identification methods using λ and those algorithms.

2 λ Fuzzy Measure

Fuzzy integrals are useful tool for global evaluation models. However, the number of parameter of fuzzy measure is large. λ fuzzy measure is one class of fuzzy measures which can be identified by interaction index λ (or ξ) and weights of individual evaluation items(Figure 1).

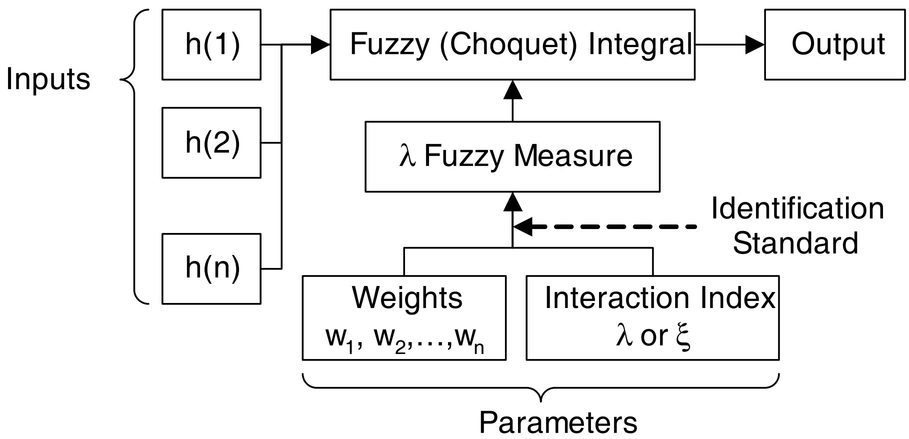

Figure 1: λ Fuzzy Measure and Fuzzy Integral

For a $\lambda > -1$, λ fuzzy measure μ_λ is defined as:

$$\mu_\lambda(A \cup B) = \mu_\lambda(A) + \mu_\lambda(B) + \lambda\mu_\lambda(A)\mu_\lambda(B) \tag{1}$$

for all $A \cap B = \emptyset$ where $\mu(X) = 1$ and $\mu(A) \in [0, 1], \forall A \in 2^X$. However, if the λ is fixed, the μ_λ is not identified uniquely. To identify the fuzzy measure uniquely, we must also specify the weights ($w_1, \ldots, w_n, w_i \geq 0$) and identification standard.

The weights of a fuzzy measure are not trivial. Therefore, 3 standards are introduced.

- Singleton Fuzzy Measure Ratio Standard
- Shapley Value Standard
- Input Number Standard

3 Singleton Fuzzy Measure Ratio Standard

This standard is to identify the fuzzy measure such that:

$$\mu_\lambda(\{1\}) : \mu_\lambda(\{2\}) : \ldots : \mu_\lambda(\{n\}) = w_1 : w_2 : \ldots : w_n \tag{2}$$

Therefore, this standard makes point of each input's single influence to the output.

Identification Algorism Varying $p \in (0, 1)$ and find $\mu(\{1, 2, \ldots, n\}) = 1$.

1. Normalise weights where $\max w_i = 1$.
2. $p := 0.5$
3. $\mu(\{i\}) := pw_i, \forall i$.
4. Calculate

   $\mu(\{1, 2, \ldots, j\}) := \mu(\{1, 2, \ldots, j-1\}) + \mu(\{j\}) + \lambda\mu(\{1, 2, \ldots, j-1\})\mu(\{j\})$

   for $j = 2, \ldots, n$
5. if $\mu(\{1, 2, \ldots, j\}) > 1$ for a j then decrease the p and go to 3.
6. if $\mu(\{1, 2, \ldots, n\}) < 1$ then increase the p and go to 3.
7. if $\mu(\{1, 2, \ldots, n\}) = 1$ then stop the algorism.

4 Shapley Value Standard

In this standard, weights are positive values. This standard is to identify the fuzzy measure μ_λ such that:

$$sh_i(\mu_\lambda) = w_i, \forall i \tag{3}$$

where $sh_i(\mu_\lambda)$ is the Shapley Value of i-th evaluation item of the fuzzy measure μ_λ. Shapley Value is defined as:

$$sh_i(\mu) = \sum_{S \subseteq X} \gamma_n(S)[\mu(S) - \mu(S \setminus \{i\})] \tag{4}$$

$$\gamma_n(S) = \frac{(n- \mid S \mid)!(\mid S \mid -1)!}{n!} \tag{5}$$

This standard make point of each input's weight.

Identification Algorism

1. Normalise weights where $\sum w_i = 1$.
2. $q_i := 1, \forall i$ (initial)
3. Identify the fuzzy measure μ_λ using Singleton Fuzzy Measure Ratio Standard where $q_1 : q_2 : \ldots : q_n = \mu_\lambda(\{1\}) : \mu_\lambda(\{2\}) : \ldots : \mu_\lambda(\{n\})$.
4. Clculate Shapley values $sh_i(\mu_\lambda), \forall i$.
5. If $sh_i(\mu_\lambda) = w_i \forall i$ then stop the algorism.
6. Find k where $w_k - sh_k(\mu_\lambda) = \max_i[w_i - sh_i(\mu_\lambda)]$.
7. $q_k := q_k + 1$
8. go to 3.

5 Input Number Standard

In this standard, weights are non-negative integers. Weights are the numbers of the same weights inputs. For example, if $n = 3$ and $w_1 = 2$, $w_2 = 4$ and $w_3 = 1$ then there are 7 same weights inputs. That is, 2 inputs are $h(1)$, 4 are $h(2)$ and one is $h(3)$ (see Figure 2).

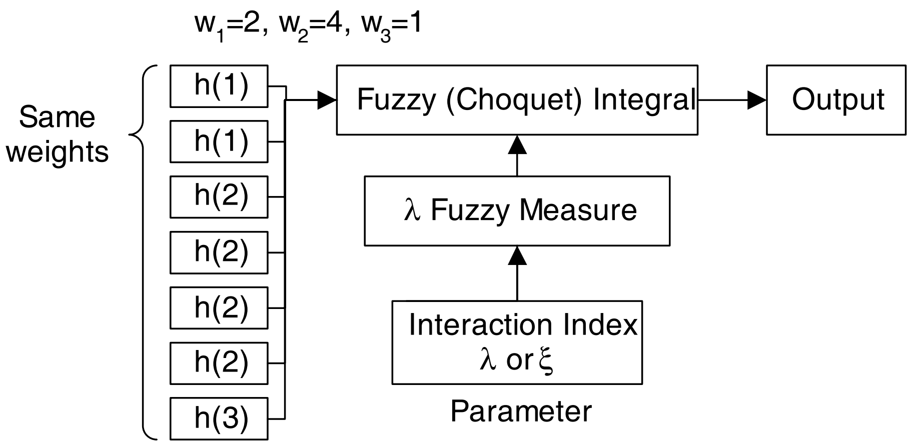

Figure 2: Input Number Standard

Identification Using ϕ_s transformation, it is easy to identify the λ fuzzy measure. u_i are normalized weights where $u_i = w_i / \sum_i w_i$.

$$\phi_s : [0, 1] \to [0, 1], \;\; s \in [0, +\infty] \tag{6}$$

$$\phi_s(u) = \begin{cases} \langle u \rangle & \text{if } s = 0 \\ u & \text{if } s = 1 \\ 1 - \langle 1-u \rangle & \text{if } s = +\infty \\ (s^u - 1)/(s-1) & \text{otherwise} \end{cases} \tag{7}$$

$$\text{where } \langle u \rangle = \begin{cases} 1 & \text{if } 0 < u \le 1 \\ 0 & \text{if } u = 0 \end{cases}$$

where $s = \lambda + 1$.

$$\mu_\lambda(A) = \phi_{\lambda+1}(\sum_{i \in A} u_i) \tag{8}$$

For example, if $\lambda = 2$ then $\mu_\lambda(\{1,3\}) = \phi_{\lambda+1}(\frac{w_1+w_3}{\sum w_i}) = \phi_3(3/7) = \frac{3^{3/7}-1}{3-1} = 0.30$.

ξ ξ is another interaction index which values $[0,1]$.

$$\xi \quad : \quad [0,\infty] \to [0,1]$$

$$\xi(s) \quad = \quad \begin{cases} 1 & \text{if } s = 0 \\ 0 & \text{if } s = +\infty \\ \frac{1}{1+\sqrt{s}} & \text{otherwise} \end{cases} \tag{9}$$

$$s = \xi^{-1}(x) = \begin{cases} +\infty & \text{if } x = 0 \\ \frac{(1-x)^2}{x^2} & \text{if } x \neq 0 \end{cases} \tag{10}$$

$\xi \in (0,1)$ has one to one correspondence with $\lambda \in (-1,\infty)$(see fig.3).

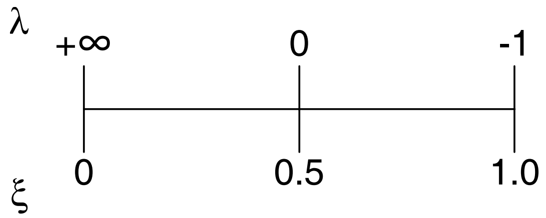

Figure 3: ξ and λ

If $\xi = 1$ then the output value of the Choquet integral is the maximum value of inputs and if $\xi = 0$ then the output value is minimum. Therefore, the input number standard makes a point of interaction index.

6 Comparison

Figure 4 is the comparison among the 3 standards. Only input number standard is an extension of maximum and minimum outputs.

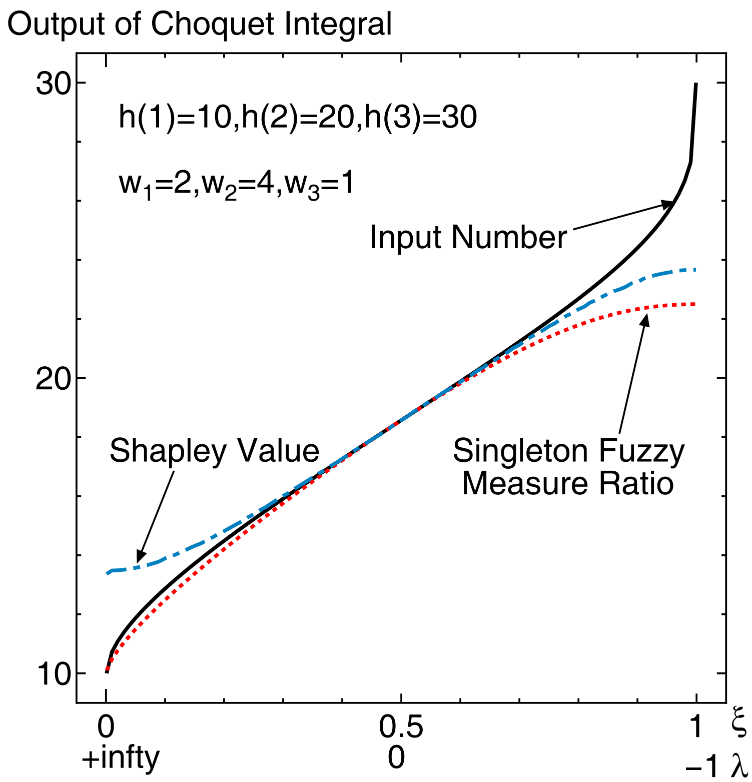

Figure 4: Comparison

References

[1] Takahagi, Eiichiro: "On Identification methods of λ-fuzzy measures using weights and λ", Japanese Journal of Fuzzy Sets and Systems, vol.12, no.5, 665-676, 2000.

[2] Takahagi, Eiichiro: "An intermediate Evaluation Method among Maximum, Average and Minimum using Fuzzy Measure and Choquet Integral and its Applications", "Methodologies for the conception, design, and application of intelligent systems; proceedings of the 4th International Conference on Soft Computing 1", "World Scientific2, 299-302, iizuka, 1996.